# Subequivariant Reinforcement Learning in 3D Multi-Entity Physical Environments

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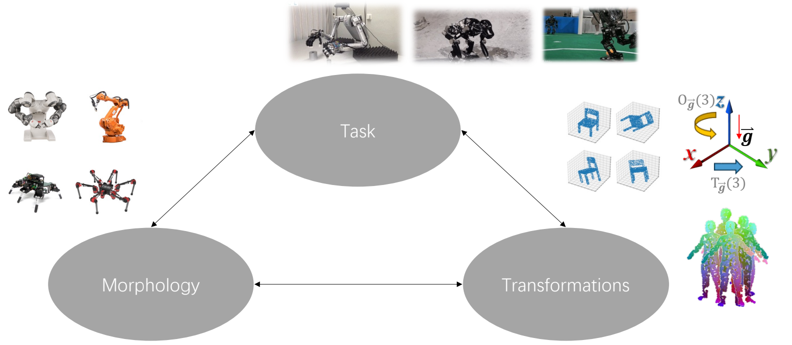
* Equal contribution

## Background

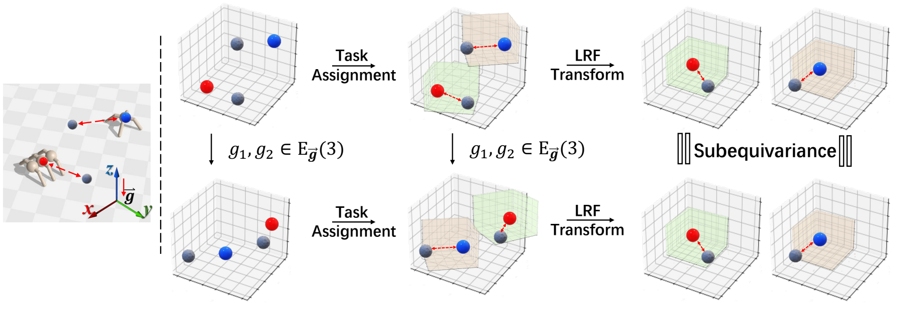
### ❑ Generalization of Embodied Agents




➢ An intricate challenge is generalizing across configurations like transformations, morphologies, and tasks, which are interlinked and complicate the learning process.

## Motivation

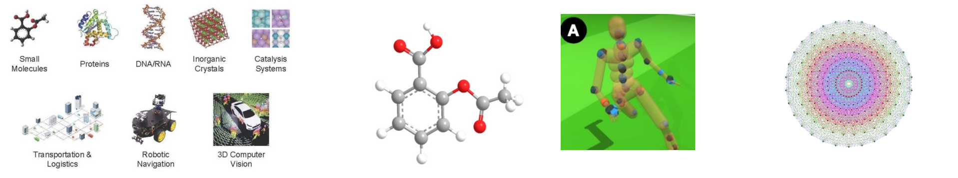
### ❑ The Symmetry in 3D Multi-Entity Physical Environments




In particular, multi-entity systems, which include agents, objects, present considerable challenges compared to single-entity scenarios, partly due to exponential expansion of global transformations as the number of entities increases.

## Spatial Intelligent

### ❑ Geometric Deep Learning




Geometric Structure and Systems | 3D Geometric Graph | Geometric Symmetry

**Geometric Symmetry** The symmetrical structure in 3D environments is $\mathrm{E}(3)$, which is a 3-dimensional Euclidean *group* that consists of rotations, reflections, and translations.

**Definition 2.1 (Group):** A group $G$ is a set of transformations with a binary operation " $\cdot$ " satisfying these properties: " $\cdot$ " is closed under associative composition, there exists an identity element, and each element must have an inverse.

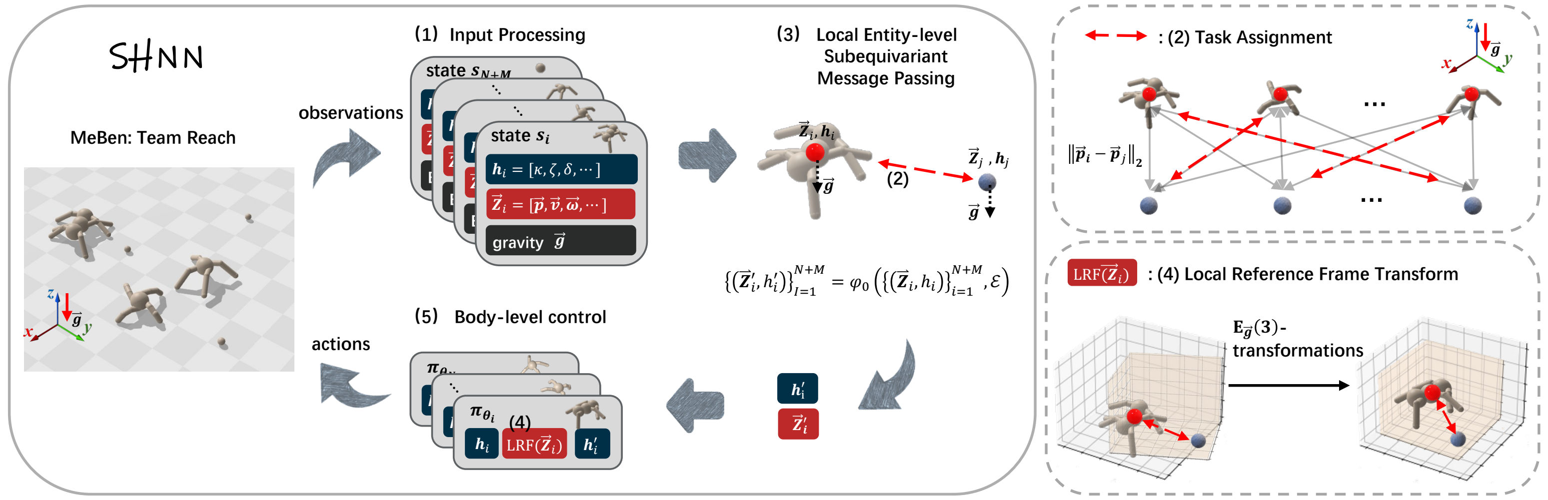
Symmetrical structure enforced on the model is formally described by the concept of *equivariance*.

**Definition 2.2 (Equivariance):** Suppose $\vec{\boldsymbol{Z}}$ to be 3D geometric vectors (positions, velocities, etc) that are steerable by a group $G$, and $\boldsymbol{h}$ non-steerable features. The function $f$ is $G$-equivariant, if for any transformation $g \in G$, $f(g \cdot \vec{\boldsymbol{Z}}, \boldsymbol{h}) = g \cdot f(\vec{\boldsymbol{Z}}, \boldsymbol{h})$, $\forall \vec{\boldsymbol{Z}} \in \mathbb{R}^{3\times m}, \boldsymbol{h} \in \mathbb{R}^d$. Similarly, $f$ is invariant if $f(g \cdot \vec{\boldsymbol{Z}}, \boldsymbol{h}) = f(\vec{\boldsymbol{Z}}, \boldsymbol{h})$.

Specifically, the $\mathrm{E}(3)$ operation " $\cdot$ " is instantiated as $g \cdot \vec{\boldsymbol{Z}} := \boldsymbol{O}\vec{\boldsymbol{Z}}$ for the orthogonal group that consists of rotations and reflections where $\boldsymbol{O} \in \mathrm{O}(3) := \{\boldsymbol{O} \in \mathbb{R}^{3\times 3} | \boldsymbol{O}^\top \boldsymbol{O} = \boldsymbol{I}\}$, and is additionally implemented as the translation $g \cdot \vec{\boldsymbol{x}} := \vec{\boldsymbol{x}} + \vec{\boldsymbol{t}}$ for the 3D coordinate vector where $\vec{\boldsymbol{t}} \in \mathrm{T}(3) := \{\vec{\boldsymbol{t}} \in \mathbb{R}^3\}$. To align with the principles of classical physics under the influence of gravity, we introduce a relaxation of the group constraint. Specifically, we consider equivariance within the subgroup of $\mathrm{E}(3)$ induced by gravity $\vec{\boldsymbol{g}} \in \mathbb{R}^3$, defined as $\mathrm{O}_{\vec{g}}(3) := \{\boldsymbol{O} \in \mathbb{R}^{3\times 3} | \boldsymbol{O}^\top \boldsymbol{O} = \boldsymbol{I}, \boldsymbol{O}\vec{\boldsymbol{g}} = \vec{\boldsymbol{g}}\}$ and $\mathrm{T}_{\vec{g}}(3) := \{\vec{\boldsymbol{t}} \in \mathbb{R}^3 | \vec{\boldsymbol{t}}\vec{\boldsymbol{g}} = \vec{\boldsymbol{0}}\}$. By this means, the $\mathrm{E}_{\vec{g}}(3)$-equivariance is only restrained to the translations/rotations/reflections along the direction of $\vec{\boldsymbol{g}}$. We term *subequivariance* primarily referring to $\mathrm{E}_{\vec{g}}(3)$-equivariance.

## Method

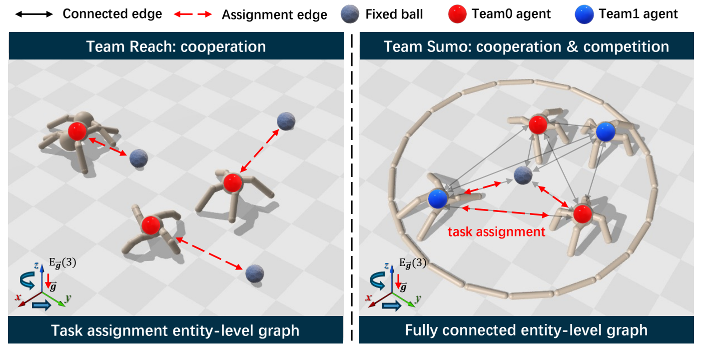
### ❑ Subequivariant Hierarchical Neural Networks (SHNN)




### ❑ Multi-entity Benchmark (MeBen)

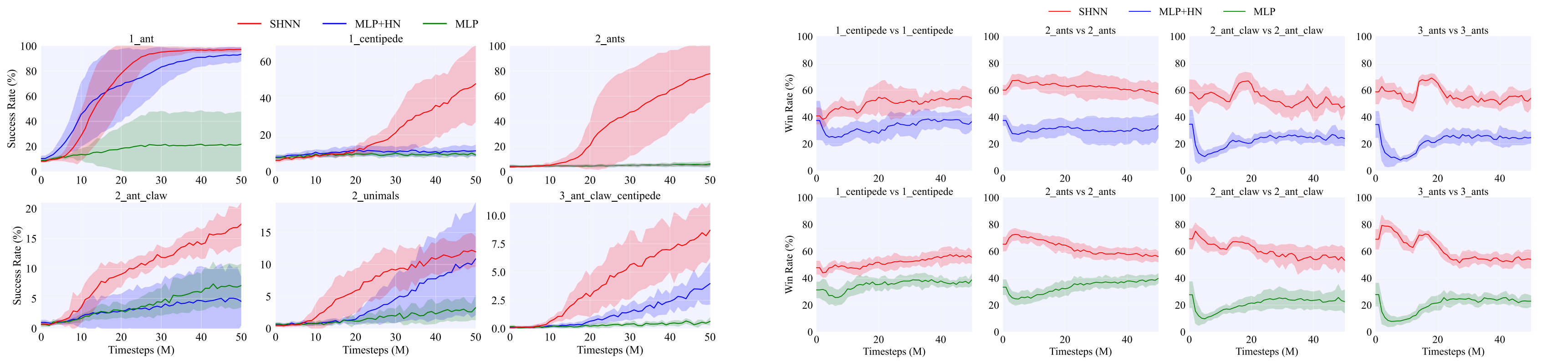
*Table 1.* Comparison of Morphology-based Environment Setup.

| Aspect | SGRL | MxT-Bench | MeBen |
|---|---|---|---|
| Multi-Morphology | ✓ | ✓ | ✓ |
| Multi-Agent | × | × | ✓ |
| Diverse-Task | × | ✓ | ✓ |
| Supported-Symmetry | ✓ | × | ✓ |
| Accelerated-Hardware | × | ✓ | ✓ |




## Experiment

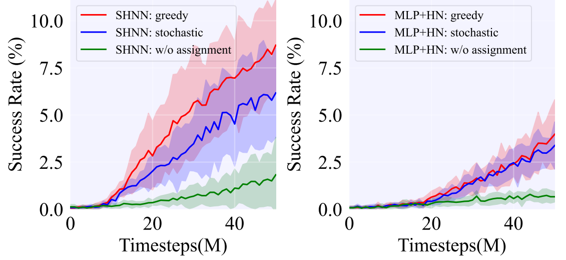
### ❑ Evaluations in Diverse Environments



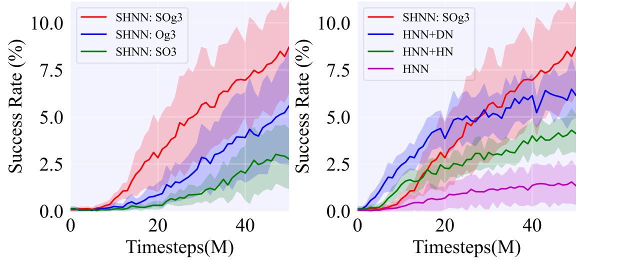

### ❑ Extended Evaluations on Transformer

| Methods | 1_ant | 1_centipede | 2_ants | 2_ant_claw | 2_unimals | 2_ant_claw_centipede |
|---|---|---|---|---|---|---|
| MLP+HN | 93.39 ± 5.25 | 11.28 ± 3.21 | 5.25 ± 1.399 | 4.52 ± 3.93 | 10.86 ± 8.74 | 3.98 ± 1.83 |
| SHNN | **97.26** ± 1.51 | **47.82** ± 20.62 | **77.93** ± 22.22 | **17.40** ± 3.54 | **11.97** ± 2.31 | **8.70** ± 2.42 |
| Transformer+HN | 5.47 ± 2.84 | 5.55 ± 2.99 | 2.47 ± 0.73 | 0.51 ± 0.19 | 0.25 ± 0.10 | 2.26 ± 1.92 |
| SHTransformer | **63.61** ± 39.57 | **11.52** ± 2.39 | **11.37** ± 15.50 | **1.17** ± 0.70 | **0.26** ± 0.15 | **7.12** ± 2.29 |

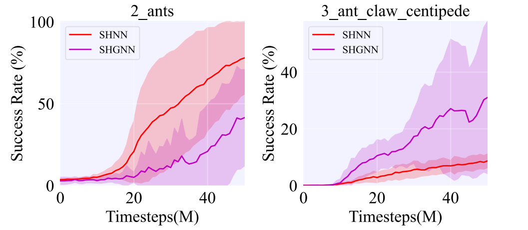
### ❑ Ablations on Assignment



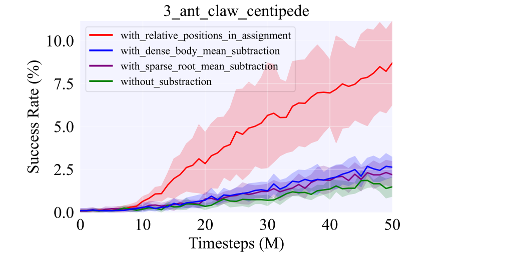

### ❑ Ablations on Equivariance



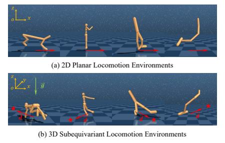

### ❑ Analyses of Morphology-shared Policy




### ❑ Importance of Local Symmetry




### Our ICML 2023 Oral



❑ https://alpc91.github.io/SGRL/

### ❑ Model Comparison: Parameters and Training Time

| Model | Parameters (M) | Training Wall Time (h) |
|---|---|---|
| MLP+HN | **1.759** | 0.299±0.003 |
| Transformer+HN | 0.416 | 1.545±0.001 |
| SHNN | 0.772 | 1.302±0.012 |
| SHTransformer | 0.711 | 2.487±0.014 |
| SHGNN | 0.613 | **5.538**±1.411 |

## Our Contributions

➢ We propose SHNN, a framework to optimize policies in 3D multi-entity environments, replacing hand-crafted LRFs with a model that isolates local transformations and compresses state space using local geometric symmetry, especially under gravity influences.

➢ Introducing MeBen, a set of MARL environments focused on morphology-based multi-entity interactions, supporting both cooperative and competitive dynamics across various transformation scenarios.

## Website

❑ https://alpc91.github.io/SMERL/
❑ https://github.com/alpc91/SMERL

## Wechat

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