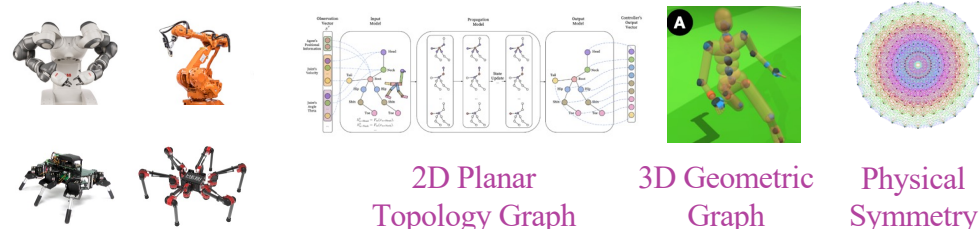


## Background

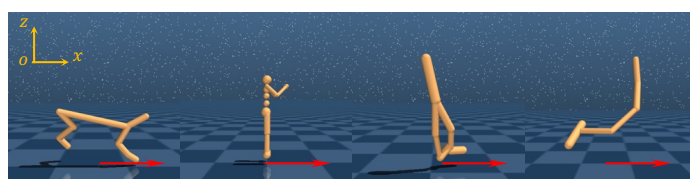
### Challenges posed by morphology



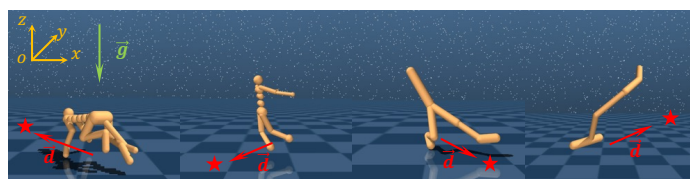
- Each robot has a different morphology.
- A separate policy is trained for each robotics setup. It doesn't generalize.
- Prior Attempts only use topology graph in 2D Planar environments.
- Real Physical World is 3D Geometric Structure and Systems, which contains Physical Symmetry.

## Motivation: 3D-SGRL

- Illustrative comparison between previous 2D planar setting and our 3D subequivariant formulation.



(a) 2D Planar Locomotion Environments



(b) 3D Subequivariant Locomotion Environments

	2D-Planar	Our 3D-SGRL
State Space	Range Initial Target	$xoz$ -plane $x^+$ -axis $x^+$ -axis
Action Space	# Actuators DoF	3 per joint 3 per joint
Symmetry	External Force Group	Gravity $\vec{g}$ , Target $\vec{d}$ $O_{\vec{g}}(3)$

## Method: SubEquivariant Transformer (SET)

### Equivariance

Definition 2.1 (E(3)-equivariance). Suppose  $\vec{Z}$  to be 3D geometric vectors (positions, velocities, etc) that are steerable by E(3) transformations, and  $\mathbf{h}$  non-steerable features.

- The function  $f$  is E(3)-equivariant, if for any transformation  $g \in E(3)$ ,  $f(g \cdot \vec{Z}, \mathbf{h}) = g \cdot f(\vec{Z}, \mathbf{h})$ ,  $\forall \vec{Z} \in \mathbb{R}^{3 \times m}$ ,  $\mathbf{h} \in \mathbb{R}^d$ .
- Similarly,  $f$  is invariant if  $f(g \cdot \vec{Z}, \mathbf{h}) = f(\vec{Z}, \mathbf{h})$ .

### SubEquivariance

Han et al. (2022a) additionally considers equivariance on the subgroup of  $O(3)$ , induced by the external force  $\vec{g} \in \mathbb{R}^3$  like gravity, defined as

$$O_{\vec{g}}(3) := \{O \in \mathbb{R}^{3 \times 3} \mid O^T O = I, O\vec{g} = \vec{g}\}$$

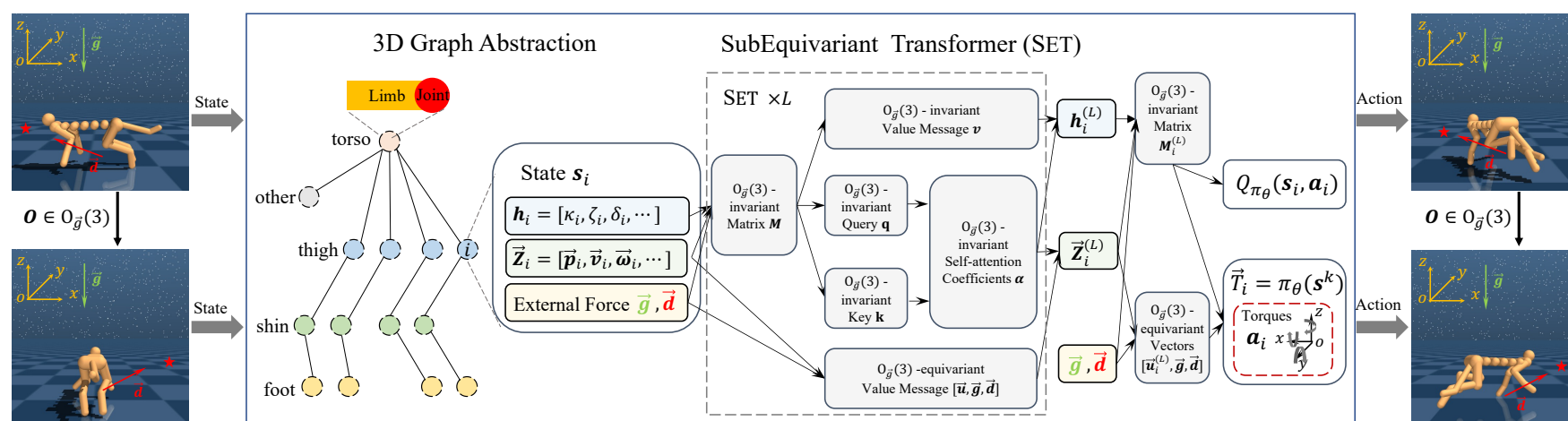
Han et al. (2022a) also presented a universally expressive construction of the  $O_{\vec{g}}(3)$ -equivariant functions:

$$f_{\vec{g}}(\vec{Z}, \mathbf{h}) = [\vec{Z}, \vec{g}] M_{\vec{g}},$$

$$\text{s.t. } M_{\vec{g}} = \sigma([\vec{Z}, \vec{g}]^T [\vec{Z}, \vec{g}], \mathbf{h}),$$

where  $\sigma(\cdot)$  is an Multi-Layer Perceptron (MLP) and  $[\vec{Z}, \vec{g}] \in \mathbb{R}^{3 \times (m+1)}$  is a stack of  $\vec{Z}$  and  $\vec{g}$  along the last dimension.

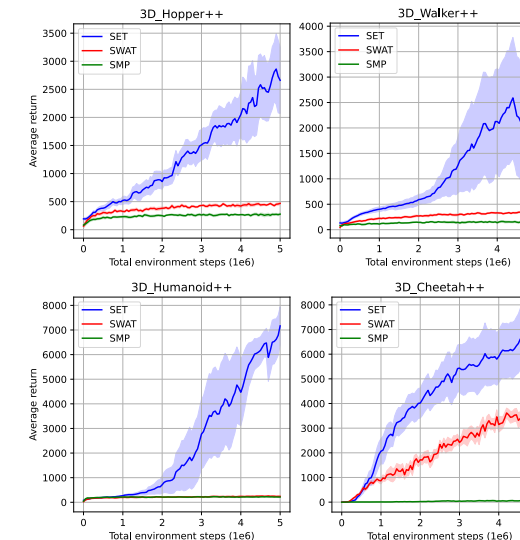
### Illustration of the Flowchart of 3D-SGRL.



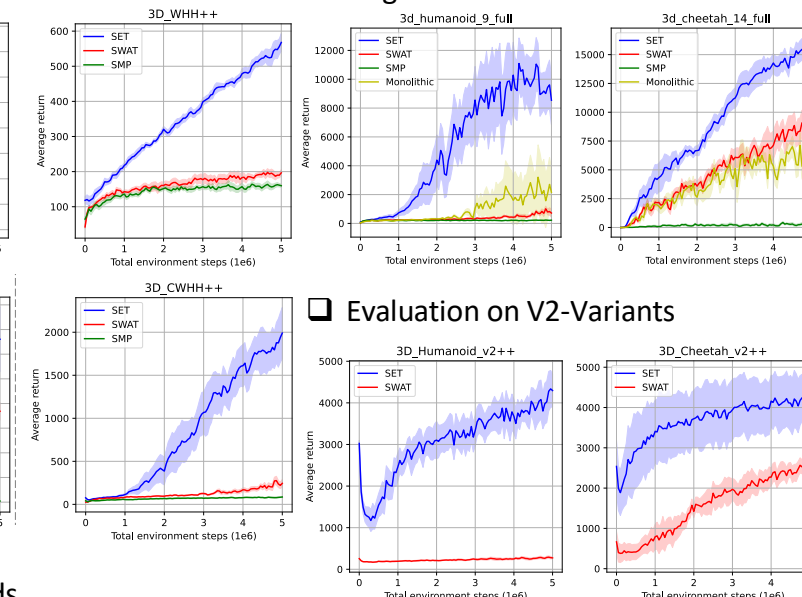
Notation:  $\vec{Z}_i \in \mathbb{R}^{3 \times 6}$  include its position  $\vec{p}_i \in \mathbb{R}^3$ , positional velocity  $\vec{v}_i \in \mathbb{R}^3$ , rotational velocity  $\vec{\omega}_i \in \mathbb{R}^3$ , etc.  $\mathbf{h}_i \in \mathbb{R}^{13}$  consist of the rotation angles  $\kappa_i, \zeta_i, \delta_i$  of joint axes, etc. External forces like gravity  $\vec{g} \in \mathbb{R}^3$  and a target direction  $\vec{d} \in \mathbb{R}^3$ . Also,  $\vec{u}_i = \vec{Z}_i \mathbf{W}_u$ , where  $\mathbf{W}_u$  is a matrix.

## Experiment

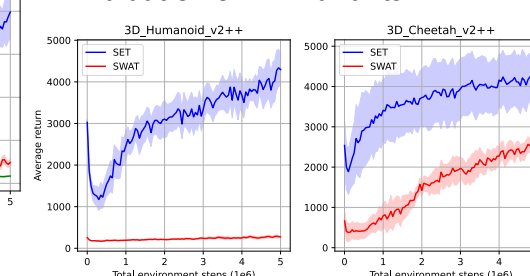
### Multi-Task with different Morphologies



### Single-Task



### Evaluation on V2-Variants

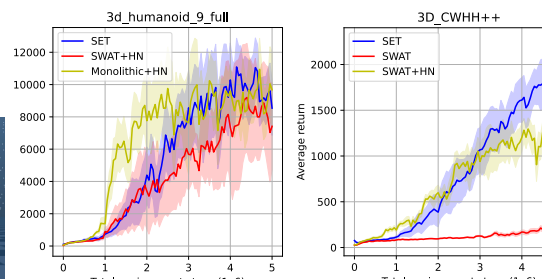


### Comparison with Invariant Methods

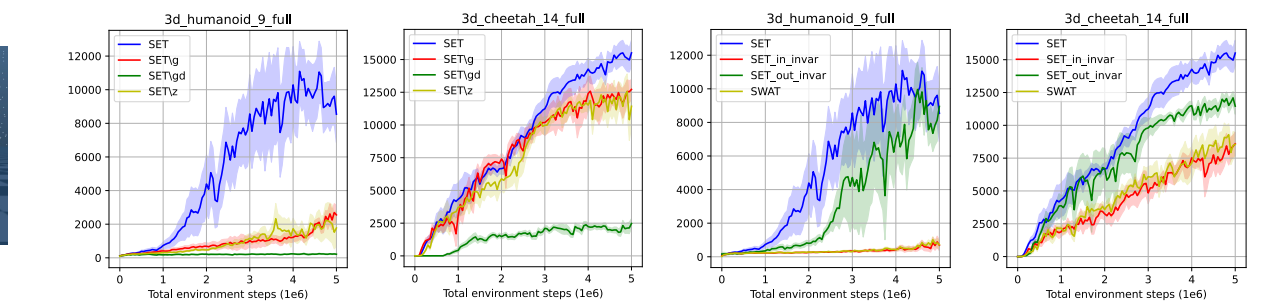
Environment	SET	SWAT+HN
cross-domain (3D_CWHH++)		
3d.walker_3	206.8 ± 37.4	26.3 ± 72.4
3d.walker_6	243.7 ± 32.3	156.8 ± 11.1
3d.humanoid_7	161.9 ± 3.4	130.2 ± 2.1
3d.humanoid_8	180.0 ± 6.5	152.9 ± 36.8
3d.cheetah_11	1078.1 ± 722.8	786.5 ± 779.3
3d.cheetah_12	3038.3 ± 2803.3	2517.3 ± 2113.9

### Zero-Shot Generalization

Environment	SET	SWAT	SMP
in-domain (3D_Walker++, 3D_Humanoid++, 3D_Cheetah++)			
3d.walker_3	276.2 ± 17.4	207.0 ± 52.7	56.8 ± 15.1
3d.walker_6	431.3 ± 146.2	358.0 ± 58.9	143.4 ± 50.7
3d.humanoid_7	244.8 ± 7.9	170.3 ± 51.7	190.9 ± 16.2
3d.humanoid_8	299.6 ± 23.7	141.4 ± 22.1	185.4 ± 9.2
3d.cheetah_11	4643.9 ± 292.6	1785.3 ± 999.3	2.0 ± 2.9
3d.cheetah_12	916.0 ± 39.7	744.1 ± 317.1	29.8 ± 10.7
cross-domain (3D_CWHH++)			
3d.walker_3	206.8 ± 37.4	17.9 ± 13.7	18.0 ± 22.9
3d.walker_6	243.7 ± 32.3	114.9 ± 40.3	103.9 ± 1.8
3d.humanoid_7	161.9 ± 3.4	152.0 ± 6.8	124.2 ± 15.7
3d.humanoid_8	180.0 ± 6.5	156.6 ± 1.7	129.3 ± 0.1
3d.cheetah_11	1078.1 ± 722.8	4.3 ± 1.6	6.2 ± 0.5
3d.cheetah_12	3038.3 ± 2803.3	349.7 ± 304.3	6.6 ± 1.2



### Ablation



## Our Contributions

- We introduce a new morphology-agnostic RL benchmark that extends the widely adopted 2D-Planar setting to 3D-SGRL, permitting significantly larger exploring space of the agents with arbitrary initial location and target direction.

- To learn a policy in this massive search space, we design SET, a novel model that preserves geometric symmetry by construction.

## Website

- <https://alpc91.github.io/SGRL/>
- <https://github.com/alpc91/SGRL>

## Wechat

